

Math in Moscow, 2013-14 academic year**Ordinary differential equations****Exercises for lesson 5 (03/17/2014)***Ilya Schurov***Problem 1.** Find Lie derivative of $F(x, y) = x^2 - y^2$ along the following vector fields:

- (a) $(2, 3)$;
- (b) (x, y) ;
- (c) (y, x) ;
- (d) $(1, -e^y)$.

Problem 2. Find the first integral and sketch phase curves:

- (a) $\dot{x} = y, \quad \dot{y} = -2x$;
- (b) $\dot{x} = y^2 - x^2, \quad \dot{y} = 2xy$;
- (c) $\dot{x} = 2y + xe^{-y}, \quad \dot{y} = e^{-y}$.

Problem 3. Prove that the following functions are first integrals of corresponding ODEs:

- (a)
$$\begin{cases} \dot{x} = y, \\ \dot{y} = -x^2 - y^2 - x, \end{cases} \quad F(x, y) = e^x \sqrt{x^2 + y^2}.$$
- (b)
$$\begin{cases} \dot{x} = x, \\ \dot{y} = x^2 + y^2 + y, \end{cases} \quad F(x, y) = x + \operatorname{arctg} \frac{x}{y}.$$

Problem 4. Are there exists any globally defined nonconstant continuous first integral for the following systems:

- (a) $\dot{x} = x, \quad \dot{y} = y$;
- (b) $\dot{x} = x, \quad \dot{y} = 2y$.