Math in Moscow, 2013-14 academic year
Ordinary differential equations
Assignment ODE-7 (To be returned 5/5/2014)

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1 (3 points each). Solve the following systems of ODE. Find the solution with initial condition \((x(0), y(0)) = (x_0, y_0)\). Draw phase portrait. Detect the type of the singular point if it is nondegenerate.

(a). \(\dot{x} = -2x - 12y, \quad \dot{y} = x + 5y\)
(b). \(\dot{x} = 5x - 6y, \quad \dot{y} = 3x - 4y\)
(c). \(\dot{x} = 2x + 2y, \quad \dot{y} = -3x - 3y\)
(d). \(\dot{x} = 5x + 4y, \quad \dot{y} = -10x - 7y\)
(e). \(\dot{x} = 2x + 3y, \quad \dot{y} = 2y\)

2 (4 + 5). Solve the following systems of ODE. Find the solution with initial condition \((x(0), y(0), z(0)) = (x_0, y_0, z_0)\).

(a). \(\dot{x} = 2x + y, \quad \dot{y} = 2y + z, \quad \dot{z} = 2z; \)
(b). \(\dot{x} = -x, \quad \dot{y} = 2x - y, \quad \dot{z} = 3x + y - z.\)

3 (6). Investigate all singular points of the system. Detect their types and stability. For nodes and saddles, find eigenvectors of the linearization. Sketch phase portrait of the system near the singular points.

\[ \dot{x} = \ln(1 - y + y^2), \quad \dot{y} = 3 - \sqrt{x^2 + 8y}. \]

4 (5). Using a theorem on stability in first approximation, find for which values of the parameter \(s\) the singular point \((0, 0)\) is asymptotically stable? For which it is Lyapunov unstable? For which \(s\) the theorem will not give the answer?

\[
\begin{cases}
\dot{x} = -4e^x + 6 \sin (y) + 4 \\
\dot{y} = -\sin (4y) + \sin (sx)
\end{cases}
\]  

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