Math in Moscow, 2013-14 academic year
Ordinary differential equations
Assignment ODE-6 (To be returned 04/14/2014)
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1 (3 points each). Solve the following ODEs:
(a). \(xy' + (x + 1)y = 3x^2e^{-x}\);
(b). \(2x(x^2 + y)dx = dy\);
(c). \((x + y^2)dy = y \, dx\);
(d). \((2e^y - x)y' = 1\);
(e). \(y' = \frac{y^2}{2y^2}\).

2 (4). Let \(x_1(t)\) and \(x_2(t)\) — two distinct solutions of nonhomogeneous linear differential equations of first order
\[ \dot{x} = a(t)x + b(t). \]
Express the solution of this equation with initial condition \(x(t_0) = x_0\) for arbitrary \(t_0\) and \(x_0\) using \(x_1(t)\) and \(x_2(t)\).

3 (3 points each). Let \(x = \varphi(t; x_0)\) be the solution of the following differential equation with initial condition \(x(0) = x_0\). Find \(\frac{\partial \varphi}{\partial x_0}\) for \(x_0 = 0\).

(a). \(\dot{x} = 2x\); (e). \(\dot{x} = t \sin x\);
(b). \(\dot{x} = \sin x\); (f). \(\dot{x} = \sin x \sin t\);
(c). \(\dot{x} = xt\); (g). \(\dot{x} = (x + x^2)t\);
(d). \(\dot{x} = x \sin t\); (h). \(\dot{x} = x^2(t + x)\).

4 (4 points each). Solve the following systems and sketch their phase portraits.

(a). \[ \begin{cases} \dot{x} = 2x + 3y \\ \dot{y} = x + 4y \end{cases} \]
(b). \[ \begin{cases} \dot{x} = 4x - 3y \\ \dot{y} = 6x - 5y \end{cases} \]
(c). \[ \begin{cases} \dot{x} = y - 6x \\ \dot{y} = y - 12x \end{cases} \]