Math in Moscow, 2013-14 academic year
Ordinary differential equations
Assignment ODE-5 (To be returned 24/03/2014)
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1 (2 + 2 + 2 + 2). For the following systems of ODE find some nonconstant globally defined continuous first or prove that it does not exists.
(a). \( \dot{x} = \sin(x + y), \quad \dot{y} = \cos(x + y + z), \quad \dot{z} = 0; \)
(b). \( \dot{x} = -y, \quad \dot{y} = x, \quad \dot{z} = \sin(x^2 + y^2 + z^2); \)
(c). \( \dot{x} = x, \quad \dot{y} = 2y, \quad \dot{z} = -3z. \)
(d). \( \dot{x} = x, \quad \dot{y} = 2y, \quad \dot{z} = 3z. \)

2 (4). For which real \( k \) there exists nonconstant globally defined continuous first integral for the system:
\[ \dot{x} = x, \quad \dot{y} = ky. \]

3 (1 + 1 + 1 + 2). Prove the following properties of Lie derivative:
(a). \( L_v(f + g) = L_vf + L_vg; \)
(b). \( L_{v+w}f = L_vf + L_wf; \)
(c). \( L_{f+g} = fL_vg; \)
(d). \( L_v(fg) = fL_vg + gL_vf. \)

4 (2 + 3). Consider equation of mathematical pendulum \( \ddot{x} = -\sin x. \)
(a). Find its first integral.
(b). Draw phase portrait. Depict all singular points (equilibrium states).