Math in Moscow, 2013-14 academic year
Ordinary differential equations
Assignment ODE-2 (To be returned 02/24/2014)
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1 (each part is 2 points). Find all solutions of an equation in form $y = y(x)$. Plot some integral curves.

(a) $y' = y/(2x)$ (b) $y' = -y/x$ (c) $y' = -4x/y$ (d) $y' = -xy$

2 (each part is 3 points). Solve the following equations (it is not needed to express the solution as a functions of x, the answer in implicit form (but without integrals) is acceptable):
(a) $y' \operatorname{ctg}^2(x) + \operatorname{tg}^2(y) = 0$;
(b) $xy' + y = y^2$;
(c) $xyy' = \sqrt{y^2 + 1}$;
(d) $\dot{x} = \sqrt{x - t} + 1$;
(e) $(x + 2y)y' = 1$. Find a solution with initial condition $y(0) = -1$;
(f) $xy' = x + y$ (hint: consider substitution $z = y/x$);
(g) $2x^3y' = y(2x^2 - y^2)$.

3 $(2 + 1)$. Let c be a positive number. A differential equation of the form

$$\frac{dy}{dt} = ky^{1+c}$$

where k is a positive constant, is called a *doomsday equation*.
(a). Determine the solution that satisfies the initial condition $y(0) = y_0$.
(b). Determine the time $t = T$ such that $\lim\limits_{t \to T^-} y(t) = +\infty$ (doomsday).

4 $(3 + 1 + 4)$. Denote by $x(t)$ the size of fish population, and assume that its normal growth rate is given by the formula $x - x^2$. Assume that we additionally fish out some fixed amount of fish in every time unit — denote this value by $c > 0$ (fishing quota). So the equation for x is the following:

$$\dot{x} = x - x^2 - c$$

(a) For which pairs of value of the fishing quota and the initial population size the population survives forever?

(b) What is the maximal value c_{max} of c for which the population survives forever if the initial population is big enough?

(c) Sketch the integral curves of corresponding differential equation for different values of c. (No exact solution needed.) Consider three cases: $0 < c < c_{max}$, $c = c_{max}$ and $c > c_{max}$.